Asset Allocation with Crypto: Application of Preferences for Positive Skewness

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Bitcoin (BTC) returns exhibit pronounced positive skewness with a third central moment of approximately 150% per year. They are well characterized by a mixture of Normals distribution with one "normal" regime and a small probability of a "bliss" regime where the price appreciation is more than 100 times at the annual horizon. The large right-tail skew induces investors with preferences for positive skewness to add significant BTC holdings to equity-bond portfolios. Even when BTC is forecast to lose half of its value in the normal regime, investors with power utility optimally add 3% allocations to BTC when the probability of the bliss regime is around 1%. Cumulative Prospect Theory investors are even more sensitive to positive skewness and hold BTC allocations of around 3% when the probability of the bliss regime is 0.0006 and the mean of BTC in the normal regime corresponds to a loss of 90%.

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"I think it can go to zero, and I think it can go to a million dollars. I have no idea."[1]

—Thomas Peterffy

Billionaire and founder of Interactive Brokers on Bitcoin

As with any new asset class, an important question is what is the optimal allocation to cryptocurrency (crypto) in a well-diversified portfolio? Thomas Peterffy, the 65th richest man in the world according to the 2021 Forbes World's Billionaire List[2] advocates holding 2-3% of one's wealth in cryptocurrencies. Ray Dalio, the second richest hedge fund manager according to the 2021 Forbes 400 list[3] and founder of Bridgewater Associates, believes that Bitcoin (BTC) "has some merit as a small portion of a portfolio" and that allocations of 1-2% to BTC are reasonable.[4]

We examine BTC returns, which is the most well-known and first crypto, over July 2010 to December 2021. Expressed as continuously compounded returns, there is large volatility of 132% per year, and extreme positive skewness. The third central moment of BTC returns is 144% at the annual horizon, which compares to -0.43% and 0.01% for equity and bond returns, respectively. We find that a mixture of Normal distributions has a close fit with the empirical distribution of BTC log returns. There is a "normal" regime with a relatively low mean (but still high in absolute terms) of 95% per year with large volatility of 114% per year. The other regime is a "bliss" regime which has an extremely high mean of 467% but with lower volatility of 51%. The bliss regime has a probability of approximately 3%. The infrequent occurrence of a draw from the bliss regime can fit the very long right-hand tail of the empirical distribution of BTC returns. In the context of Peterffy's quote, the possible BTC appreciation to "a million dollars" can be modeled by the bliss regime.

We examine the implications of the extreme positive skewness of BTC for asset allocation. We consider the work-horse power utility function (or Constant Relative Risk Aversion utility). Power utility is locally mean-variance. Yet, a power utility investor would still allocate meaningful amounts to BTC starting from a standard equity-bond portfolio—even when the unconditional mean of BTC returns is set to be negative. At the calibrated point estimates of the mixture of Normals distribution fitted to historical

[1] Carpenter, S., and C. Ballentine, "Billionaires are Embracing Crypto in Case Money 'Goes to Hell'," Bloomberg, January 1, 2022.
[2] See https://www.forbes.com/billionaires/
[3] See https://www.forbes.com/forbes-400/
[4] Locke, T., "Ray Dalio: Allocating up to 2% of Your Portfolio to Bitcoin is Reasonable," CNBC, January 5, 2022.

data, a power utility investor starting from a 60-40 equity-bond portfolio would wish to allocate 85% of their portfolio to BTC, holding the remaining 15% of their portfolio in the same fixed 60-40 equity-bond mix. We show that this is theoretically driven by the enormous third central moment of the BTC distribution, which is well captured by the bliss regime. Even when we set the mean of the normal regime distribution to correspond to a loss of 50% in the price of BTC, a 2% or 3% allocation to BTC favored by Peterffy or Dalio can be justified by small probabilities of 0.005 or 0.019, respectively, of the bliss regime occurring.

Because there are many deviations from expected utility in individuals' actual behavior, we also consider the seminal behavioral utility function of Cumulative Prospect Theory (CPT) of Kahneman and Tversky (1979) and Tversky and Kahneman (1997).[5] There are two effects driving the preference towards positive skewness as noted by Ebert and Karehnke (2019). First, the value function of CPT is defined over losses and gains, and the curvatures lead investors to prefer positively skewed payoffs. The shape of the CPT value function is actually the same as power utility in the domain of gains. The second effect is more powerful. Under CPT, investors overweight events with small probability and underweight events that occur more frequently. In the case of the BTC mixture of Normals distribution, there is a small objective probability to receive a very large payoff in the bliss regime. CPT agents overweight this outcome, and thus the optimal allocations to BTC are much more aggressive than the case of power utility.

Calibrating to Tversky and Kahneman's (1997) original CPT parameters, the optimal equity holding of equities for a CPT agent in a two-asset equity-bond portfolio is 28%. Introducing BTC in a three-asset model, the CPT investor wishes to hold a $+\infty$ position in BTC because of its extreme positive skewness. To obtain finite solutions, we change the mean of continuously compounded BTC returns to correspond to a 90% loss in the normal regime. With just a tiny probability of 0.001 of the bliss regime, the optimal BTC allocation would be approximately 10% holding the 28-72 equity-bond portfolio in the same fixed proportion in the remaining 90% of the portfolio. Optimal BTC allocations of 2% or 3% correspond to probabilities of the bliss regime around 0.0005.

It is notable that BTC returns are positively skewed, whereas negative skewness is a common property of many risky asset classes: equities (Rietz, 1988; Barro, 2009), foreign exchange (Brunnermeier, Nagel, and Pedersen, 2008), option markets (Bakshi, Kapadia, and Madan, 2009; Carr and Wu, 2007),

[5] In fact, Dertwinkel-Kalt and Koster (2020) make a case for CPT utility to explain the behavior of agents with skewness experiments. Barberis and Huang (2008) note that CPT investors have a preference for positive skewness and they argue that this can cause positively skewed assets to be overpriced and exhibit low average returns. Boyer, Mitton, and Vorkink (2010) find some empirical evidence along these lines. Our asset allocation analysis takes the traditional partial equilibrium approach: we assume the skewness and other moments of BTC, stocks, and bonds as given and investigate the implications for optimal portfolio choice.

hedge funds (Cremers, Kritzman, and Page, 2005), high yield bonds (Domian and Reichenstein, 2008), and real estate (An, Wu, and Wu, 2016; Hutson and Stevenson, 2020) are all negatively skewed. Negative skewness is also a priced risk factor in asset returns; more negatively skewed stocks are riskier and investors are compensated with higher returns.[6] A number of well-known style factor strategies are also negatively skewed like momentum (Daniel and Moskowitz, 2016) and other long-short style factor strategies (see Khandali and Lo, 2007; Ang, 2008), merger arbitrage (Mitchell, Pulvino, and Stafford, 2004), carry (Daniel, Hodrick, and Lu, 2017; Bekaert and Panayotov, 2020), and short volatility strategies (Bhansali and Harris, 2018). Baltas and Salinas (2020) show that negative skewness premium is also observed in a range of asset classes including equity indexes, government bonds, currencies, and commodities. Thus, the extreme positive skewness makes BTC a unique asset.

There is an exploding literature on crypto, and we can cite just a few papers on several related topics that we fail to capture in our model. We consider only the role of BTC as a direct investible asset. This ignores any perspectives of the remarkable technology in the blockchain or the wider web3 ecosystem of which crypto is part (see Yaga et al., 2018), the possible role of crypto as a form of fiat money and possible seigniorage value (see Gorton and Zhang, 2021), the market structure of BTC itself (Makarov and Schoar, 2021), the positive network externalities of BTC and other crypto in tokens and other membership-related features (see Sockin and Xiong, 2020), or underlying fundamental valuation of BTC (see Michnick and Athey, 2018; Cong, Li, and Wang, 2021). These effects would likely lead to further demand for BTC holdings beyond that captured by financial wealth in a utility function. On the other hand, there is large energy consumption required for Bitcoin validation, which is often supplied with non-renewable energy sources (see Stoll, Klaassen, and Gallersdorfer, 2019).

Our paper is most related to the well-established literature examining the effect of higher moments on asset allocation. Markowitz (1952) recognized early the importance of positive skewness: “the third moment of the probability distribution of returns from the portfolio may be connected with a propensity to gamble.” Early authors researching skewness and other higher moment preference are Tsiang (1972) and Kraus and Litzenberger (1976). Since then, asset allocation studies with higher moments include Ang and Bekaert (2002), Jondeau and Rockinger (2006), Guidolin and Timmermann (2008), Harvey et al. (2010), and Chiu (2010). These papers do not examine BTC allocations. There is a much smaller, but rapidly growing literature on asset allocation with BTC, or characterizing the returns of BTC or crypto, showing, among other things, they are lowly correlated to traditional assets. Among the many recent studies along these lines are Briere, Oosterlinck, and Szafarz (2015), Liu and Tsyvinski (2018), Botte and Nigro (2021),

[6] See, among others, Harvey and Siddique (2000), Ang, Chen, and Xing (2006), Boyer, Mitton, and Vorkink (2010), Xing, Zhang, and Zhao (2010), and Conrad, Dittmar, and Ghysels (2012).

and Petukhina et al. (2021). These papers generally work in a mean-variance environment and do not explicitly consider the effect of skewness for optimal portfolio choice.

The rest of this paper is organized as follows. Section 1 describes the BTC, equity, and bond return data and fits a mixture of Normals distribution to BTC returns. Section 2 examines optimal allocations to BTC for power utility, and Section 3 for behavioral CPT utility. With each utility function, we highlight the effect of the probability of the bliss regime on optimal BTC holdings. Section 4 concludes.

1. BTC Returns

We describe the BTC return data in Section 1.1. Section 1.2 fits a mixture of Normals distribution to BTC returns.

1.1 Data

We take BTC prices from July 2010 to December 2021 at the monthly frequency. For equities and bonds, we use the S&P 500 and the Bloomberg Barclays Treasury Index, respectively, from January 1973 to December 2021. Exhibit 1, Panel A graphs the cumulated returns of BTC, equities, and bonds from investing $1 at the beginning of January 1973 to the end of December 2021. The cumulated returns of BTC are staggering, reaching 772,227 at December 2021. For comparison, the cumulated returns for stocks and bonds are 4.33 and 1.35, respectively.

Because of the extremely large volatility of BTC, we work with continuously compounded, or log, returns. We compute continuously compounded returns at the annual horizon, which allows our return estimates to be easily interpreted. Setting the annual horizon also allows us to abstract from issues of time aggregation where higher moment estimates, like volatilities and skewness, can depend on the sampling frequency (see Neuberger, 2012; Neuberger and Payne, 2021).

We report summary statistics of BTC, equities, and bonds in Panel B of Exhibit 1. BTC has an annualized mean of 108.5% per year, which is 14 times larger than the mean of equities which is 7.7% per year. The standard deviation of BTC log returns is even larger, at 132.2%. In arithmetic terms, the gross expected return implied by these numbers is staggering, $\exp\left(1.085 + \frac{1}{2}(1.322)^2\right) = 7.09$. Even setting the mean to zero results in large gross returns of $\exp\left(\frac{1}{2}(1.322)^2\right) = 2.40$, which is an order of magnitude larger than the arithmetic returns in venture capital and small or micro cap stocks (see Cochrane, 2005).

While the Sharpe ratios of BTC, equities, and bonds are roughly comparable at 0.82, 0.48, and 1.17, respectively, BTC has an extremely large third central moment of 1.439, which is several orders of magnitude larger than the third central moment for stocks (-0.004) and bonds (0.000). Because the volatility of BTC is so large, the normalized third moment, or skewness metric, of 0.630 is approximately similar to bonds. However, it is the third central moment that is relevant for portfolio choice, as we show below. In the rest of the paper, we interchangeably refer to "skewness" as the third central moment, except in a few contexts where it is clear that skewness refers to the normalized third central moment, which divides the third central moment by σ^3.

Panel B of Exhibit 1 reports that BTC returns are positively correlated with equity returns, at 0.411, and negatively correlated with bond returns, at -0.300. Over this period, equities and bonds exhibit little correlation. We build in these correlations into our data generating processes in our optimal allocation with and without BTC.

In Exhibit 2, we graph kernel estimates of probability density functions (PDFs) of BTC, stocks, and bonds using the full samples of each univariate series. Panel A shows the relatively low volatilities of stocks and bonds manifests as very high and narrow PDFs. The negative skew of stocks (normalized skewness of -1.042) is visible in the blue left-hand tail. The support of the PDF for BTC is extremely wide from below -2 to above 6. Recall that these are continuously compounded returns, so the arithmetic returns range from losing everything to multiplying an initial position by hundreds.

1.2 Mixture of Normals

To fit the positive skewness of BTC returns, we fit a simple mixture of Normals distribution. These have been commonly used in economics and finance to capture higher moments and they are also extensively used in classification and clustering models in machine learning.[7] The mixture of Normals can be easily estimated by maximum likelihood or EM algorithms (see Hamilton, 1994).

We estimate the following process for BTC returns, r_{BTC}:

$$r_{BTC} = \begin{cases} N(\mu_1, \sigma_1^2) \text{ with probability } p \\ N(\mu_2, \sigma_2^2) \text{ with probability } 1 - p \end{cases} \quad (1)$$

where $N(\mu_1, \sigma_1^2)$ is the Normal distribution with mean μ_1 and variance σ_1^2 in the "bliss" regime which is drawn with probability *p*. We refer to this regime as "bliss" because we assign the high mean to regime 1.

[7] See Ang and Timmermann (2012) for a literature review in finance and economics, and the textbook by McLachlan and Peel (2000) for machine learning applications.

The second "normal" regime occurs with probability $1-p$ with distribution $N(\mu_2, \sigma_2^2)$ with mean μ_2 and variance σ_2^2.

In Exhibit 3, Panel A, we graph the estimated probability density function of the mixture of Normals in equation (1) along with the empirical kernel estimate of the density function (which is repeated from Exhibit 2 but plotted on a different scale). The right-hand tail is visible in Panel A, and there is a close fit between the estimated mixture of Normals and the empirical distribution.

In Panel B, we graph the Normal distributions in each regime. These correspond to the estimated parameters listed in Panel C of equation (1). The first regime is a bliss regime with a continuously compounded mean of μ_1 = 4.676 per year and standard deviation $\sigma_1 = 0.506$ per year. The spectacular returns that occur in the bliss regime occur with probability *p* = 0.036. The bliss regime is responsible for the large positive skewness that is observed in the PDF graph in Panel B. The second regime closely corresponds to the unconditional kernel estimated density—without the extreme right-hand tail for which the high-mean bliss regime is responsible. Most of the time, with probability $1-p=0.964$, BTC returns are drawn from a Normal distribution with mean μ_2 = 0.950 per year and standard deviation $\sigma_2 = 1.135$ per year. Note because the standard deviation is *larger* than the mean, it is highly likely that a negative return occurs in the normal regime. In fact, $\sigma_2 = 1.135$ is approximately 7 times larger than the unconditional equity standard deviation, which is 0.161 (see Exhibit 1).

It is interesting that the regime occurring infrequently has a relatively *high* mean and the normal regime occurring most of the time has a relatively *low* mean. This is opposite to most mixture of Normal estimations (or more general Markov regime-switching model estimations popularized by Hamilton, 1989) with risky asset returns like equities, as summarized by Ang and Timmermann (2012). Intuitively, stock returns exhibit negative skewness and a regime with infrequent crashes, or a regime with a negative mean, fits this pattern well. In contrast, BTC returns have pronounced positive skewness, so we occasionally draw from a regime with very high returns.

We compute the implied third central moment and skewness statistics from the mixture of Normals distribution. These are given by the following formula for the unconditional variance, σ^2:

$$\sigma^2 = p\sigma_1^2 + (1-p)\sigma_2^2 + p(1-p)(\mu_1 - \mu_2)^2, \tag{2}$$

and for the third central moment by:

$$E[(r_{BTC} - \mu)^3] = p(1-p)(\mu_1 - \mu_2)^2\{3(\sigma_1^2 - \sigma_2^2) + (1-2p)(\mu_2 - \mu_1^2)^2\}, \tag{3}$$

with the unconditional mean given by $\mu = p\mu_1 + (1-p)\mu_2$. (See also Timmermann, 2000.) In equations (2) and (3), the difference in means between each regime contributes to both the unconditional variance and the third central moment. The switching Normal distribution has an implied third central moment of 1.270, which is close to the empirical estimate of 1.439 (see Exhibit 1). The standardized skewness is also closely matched, with an implied model estimate of 0.556 compared to the data estimate of 0.630 (see Exhibit 1). The fitted mixture of Normals process slightly underestimates the third central moment and skewness, which means that the results we present below for optimal BTC allocation will be conservative in the sense that increasing skewness would only further increase the optimal BTC allocations.

2. Power Utility

This section examines optimal holdings of BTC, equity, and bonds with power utility. Section 2.1 details the solution method. We examine the effect of skewness theoretically in Section 2.2 and empirically in Section 2.3 for the fitted mixture of Normals distribution for BTC. In Section 2.4, we present a hypothetical exercise to infer an investor's belief that BTC will be drawn from the bliss regime with very high returns.

2.1 Optimal Allocations

Denote $r = \left(r_{BTC}\ r_{eq}\ r_{bd}\right)$ as the vector of continuously compounded returns on BTC, equities, and bonds, respectively. We assume that r_{BTC} follows the mixture of Normals process given in equation (1) with the parameters reported in Exhibit 3. So that we focus only on the skew induced by BTC, we assume that continuously compounded returns of equities and bonds, r_{eq} and r_{bd}, respectively, are log normal with the full sample moments in Exhibit 1. In each regime of bliss or normal returns, we set the correlation structure of BTC, equity, and bond returns to the empirical estimates listed in Exhibit 1.

Our problem is to maximize expected utility:[8]

$$\max_h E[U(W)], \tag{4}$$

[8] Our solution method is to solve the first order conditions of equation (4) using Gaussian-Hermite quadrature. This is extremely accurate for Normal distributions with as few as 4 or 5 quadrature points (see Kochenderfer and Wheeler, 2019).

over the holdings of BTC, equities, and bonds, $h = (h_{BTC}\ h_{eq}\ h_{bd})$, respectively. We consider asset allocation problems with just equities and bonds $(h_{eq}\ h_{bd})$ before considering the full allocation problem which adds the holdings of BTC, h_{BTC}.

In this section, we consider power utility (also called Constant Relative Risk Aversion, CRRA, utility) over end-of-period wealth, W:

$$U(W) = \frac{W^{1-\gamma}}{1-\gamma}, \tag{5}$$

where γ is the coefficient of risk aversion. As $\gamma \to 1$ in equation (5), we have log utility, $U(W) = \log(W)$.

Wealth, W, is given by:

$$W = h_{BTC} \exp(r_{BTC}) + (1 - h_{BTC})\left[h_{eq} \exp(r_{eq}) + h_{bd} \exp(r_{bd})\right]. \tag{6}$$

We have written wealth in equation (6) so that the equity-bond portfolio has weights h_{eq} and h_{bd} in equities and bonds, respectively, that sum to one.[9] We do this because of the prevalence of the 60-40 equity-bond portfolio (or other equity-bond mixes). In our empirical work, we calibrate the risk aversion parameter, γ, to give a particular equity-bond mix like 60-40 in a two-asset problem of just equities and bonds. Then, we hold that level of risk aversion fixed to solve for the optimal bitcoin holding in the optimization (equation (4)) with three assets and hold the equity-bond mix constant. Another way of saying this is that when we introduce BTC, the BTC holding is funded by pro-rata positions in equities and bonds.

2.2 Effect of Skewness in Theory

Following Samuelson (1970), we can expand the expectation of a utility function, U, as a Taylor series around the point $\bar{W}$ for average wealth as:

$$E[U(W)] \approx U(\bar{W}) + U'(\bar{W})E[W - \bar{W}] + \frac{1}{2}U''(\bar{W})E[(W - \bar{W})^2] + \frac{1}{3!}U'''(\bar{W})E[(W - \bar{W})^3], \tag{7}$$

[9] See Lawler et al. (2020) for comments on 60-40 equity-bond portfolios in a retail setting. For institutions, these equity-bond portfolios are often called "reference portfolios" (see Ang, Brandt, and Denison, 2014).

stopping at the third moment with U', U'', and U''' are the first, second, and third derivatives, respectively, of the utility function, U. Scott and Horvath (1980) show that for well-behaved utility functions, investors like moments with odd powers, like the mean and skewness, and dislike moments with even powers, like the variance.[10] Taking expectations, the second term in equation (7), $U'(\bar{W})E[W-\bar{W}]$, drops out.

Define the certainty equivalent as $W_{CE} = U^{-1}(\,E[U(W)])$, which is the certain amount we are willing to pay to have the same expected utility of an uncertain wealth outcome, and take a linear expansion of the certainty equivalent utiliy as $U(W_{CE}) \approx U(\bar{W}) + U'(\bar{W})(W_{CE} - \bar{W})$. Since $U(W_{CE}) = E[U(W)]$ and $E[W] = \bar{W}$, we can equate the linear expansion of the certainty equivalent with equation (7) to yield:

$$U'(\bar{W})(E[W] - W_{CE}) \approx \frac{1}{2}U''(\bar{W})\sigma^2 - \frac{1}{6}U'''(\bar{W})E[(W-\bar{W})^3], \tag{8}$$

where $\sigma^2 = E[(W-\bar{W})^2]$ is the variance. The risk premium, $\pi = E[W] - W_{CE}$, can be written:

$$\pi = E[W] - W_{CE} \approx \frac{1}{2}\gamma\sigma^2 - \frac{1}{6}\gamma\eta\, E[(W-\bar{W})^3], \tag{9}$$

where

$$\gamma = {}^{U''W}\!/_{U'}$$

is the Arrow-Pratt measure of relative risk aversion and

$$\eta = {}^{U'''}\!/_{U''}$$

is the prudence parameter of Kimball (1990). Equation (9) states the investors demand compensation for bearing variance risk, through the relative risk aversion coefficient, γ, and that they are willing to pay to bear positive skewness risk as represented through the third central moment, $E[(W-\bar{W})^3]$. Put another way, agents dislike variance risk but like positive exposure to third moment risk. Both the risk aversion

[10] Jondeau and Rockinger (2006) use the Taylor series expansion of expected utility up to fourth moments for asset allocation with exponential utility and Harvey et al. (2010) use up to third moments in a Bayesian setting. Of course, the Taylor expansion, even to infinity, may not uniquely pin down the utility function. As Loistl (1976) notes, there are many examples where moment preferences from Taylor series will not converge or not map into preferences. We avoid this by dealing directly with utility functions.

coefficient, γ, and the prudence coefficient, η, affect the trade-off of third moment risk and return. We emphasize it is the third central moment, not normalized skewness which divides by σ^3, that is important.

The ubiquitous mean-variance utility ignores the third moment term and other higher moments, because they are usually very small. In fact, power utility, $U(W) = W^{1-\gamma}/(1-\gamma)$, is locally mean-variance utility for "small" gambles (in the sense of Pratt, 1964). This is not because risk aversion or prudence are small: for power utility, the coefficient on the third central moment is $\frac{1}{6}\gamma\eta = \frac{1}{6}\gamma(\gamma+1)$. Instead, it is because the higher central moments are usually negligible. For example, the third central moment estimates for equities and bonds are -0.004 and 0.000, respectively (as reported in Exhibit 1, Panel B). The third central moment, in contrast, for BTC is enormous at 1.439. In fact, the skewness of BTC is so large that the risk premium equation (9) is negative for BTC! For example, if $\gamma = 2$ and using the empirical estimates for the variance and third central moment for BTC reported in Panel B of Exhibit 1, the risk premium is -113%. Thus, even under power utility, which is locally mean-variance, we expect significant allocations to BTC because of its positive skew. We now show this is the case.

2.3 Effect of Skewness in Practice

We perform the following asset allocation exercise. Suppose we start with a 60-40 equity-bond portfolio. Assuming that equity and bond returns are log normal, fitted to the moments reported in data in Exhibit 1, we compute the power utility risk aversion coefficient to yield that 60-40 allocation. In this case, a risk aversion of $\gamma = 1.50$ corresponds to 60-40. Then, we hold the risk aversion fixed and estimate the optimal weight of BTC, h_{BTC}, assuming that the equity and bond weights are held in the same pro-rata 60-40 allocation. That is, the equity and bond weights are set to be $(1-h_{BTC})h_{eq}$ and $(1-h_{BTC})h_{eq}$, respectively. We use the mixture of Normals distribution (see equation (1) and Exhibit 3) for BTC returns that captures large positive skewness.

Exhibit 4 presents the results. Starting with a 60-40 equity-bond portfolio, which is produced with a risk aversion of $\gamma = 1.50$, the optimal BTC allocation is a large 84.9%! The remainder of the portfolio, 15.1% is split 60-40 between equities and bonds. Although BTC has an extremely large volatility of 1.322 (see Exhibit 1), the pronounced positive skewness leads to large allocations and dominates in the utility function (see equation (9)). The certainty equivalent compensation required to *not* invest in BTC is close to 200%. In Exhibit 4, starting at a 20-80 stock-bond portfolio leads to a 12.5% BTC allocation, and a risk tolerant investor with an 80-20 stock-bond portfolio desires a levered position in BTC of 106.6%.

2.4 Varying the Probability of Bliss

In this section we perform a revealed preference exercise and to back out an investor's beliefs on the probability of the bliss regime for small allocations to BTC. In this exercise, we deliberately set the mean of BTC in the normal regime to correspond to losing half an investor's wealth, so $\exp(\mu_2) = 0.5$, with $\mu_2 = -0.693$. We hold all other parameters of the mixture of Normals distribution constant at their values in Exhibit 3. By assuming a significantly negative unconditional mean, any allocation to BTC must be a result of positive skewness! We capture that skewness by the small probability of drawing from the very high return regime, and we investigate what probability is required to hold a given position of BTC.

Exhibit 5 presents the results. We assume that the equity-bond mix is fixed at 60-40 and use the corresponding risk aversion of $\gamma = 1.50$. For a given probability of the bliss regime, p, Exhibit 5 graphs the optimal BTC holding, h_{BTC}. The equity and bond weights in the three-asset (BTC, equity, and bonds) portfolio are $(1 - h_{BTC}) \times 0.6$ and $(1 - h_{BTC}) \times 0.4$, respectively. In Exhibit 5, Panel A, a 1% holding of BTC corresponds to a probability of the bliss regime of $p = 0.0003$. For $h_{BTC} = 2\%$, $p = 0.0052$ and for $h_{BTC} = 3\%$, $p = 0.0193$. The left-most point on the graph corresponds to $p = 0.0001$, where the BTC holding is 0.92%.[11]

In Panel B of Exhibit 5, we graph some implied statistics as a function of the probability of the bliss regime. At the top, we graph the implied unconditional BTC mean, which is given by $\mu_{BTC} = p\mu_1 + (1 - p)\mu_2$, with μ_1 and μ_2 set at the mixture of Normals estimate for the bliss regime in Exhibit 3: $\mu_1 = 4.676$ and $\mu_2 = -0.693$. By construction, the unconditional BTC mean increases as p increases, but the means are all negative with positive BTC weights. Confirming the theoretical intuition in equation (9), it is the large positive central third moment, plotted in the middle panel, which is driving this result—even for locally mean-variance power utility! The last panel graphs the probability of losing money in the normal regime, $p(r_{BTC} < 0 \,|\, \text{regime} = 2)$ which varies from 70% to 76% for this range of p. Clearly, only extremely small probabilities of the bliss state are necessary to create small BTC positions in a diversified portfolio—even when the BTC returns have large unconditional negative means and large probabilities of experiencing losses.

[11] Of course, when $p = 0$, the bliss regime is a "nuisance" distribution (see Hansen, 1996) and given the negative μ_2 parameter in this exercise, the optimal BTC holding is zero.

3. Cumulative Prospect Theory

In this section, we investigate BTC allocation with Cumulative Prospect Theory (CPT), or loss aversion, preferences. We describe CPT utility in Section 3.1. In Section 3.2, we examine optimal BTC allocations under these behavioral preferences.

3.1 Parameterization

Cumulative Prospect Theory as originally formulated in Kahneman and Tversky (1979) and extended in Tversky and Kahneman (1992) has had profound influence in the fields of economics, psychology, and other behavioral sciences.[12] There are four parts of CPT: (1) the utility is measured in terms of gains or losses relative to a reference point, (2) loss aversion, where investors are more sensitive to losses than they are to gains of the same magnitude, (3) the value function is concave over gains and convex over losses, and (4) investors perform a reweighting of objective probabilities.

Value Function

Exhibit 6, Panel A graphs the value function of CPT, which is given by:

$$v(x) = \begin{cases} x^{\gamma_1} & \text{for } x \geq 0 \\ -\lambda(-x)^{\gamma_2} & \text{for } x < 0 \end{cases} \tag{10}$$

where x is the gain or loss relative to a reference point. We set the reference point to be zero reflecting "the status quo" following Kahneman and Tversky (1979). Although the value function uses the power value function, it is defined relative to the reference point so that there is a concave shape in the domain of gains, which is associated with risk-averse behavior over gains. The utility gains level off as gains become large. The value function takes a convex shape for $x < 0$, which reflects risk-seeking behavior over losses. The utility levels off as losses increase, which can be interpreted as "bad things lead to psychic numbing" (Weber and Johnson, 2009) The risk-seeking behavior implies that individuals would prefer to gamble with the possibility of a loss, rather than endure a sure loss than is smaller than the expected value of the gamble.

We use Tversky and Kahneman's (1992) parameter values throughout this section, which other authors have shown are not too dissimilar from empirical estimates from laboratory studies or in the real

[12] A literature review celebrating 30 years after the seminal loss aversion paper is by Barberis (2013).

world (see, for example, Booij, van Praag, and van de Kuilen, 2010; Rieger, Wang, and Hens, 2017).[13] The loss aversion parameter λ governs how much more investors weight losses than gains and we set $\lambda = 2.25$. Likewise, we take Tversky and Kahneman's parameters for $\gamma = \gamma_1 = \gamma_2 = 0.88$, which control the curvature of the risk-seeking (risk-averse) behavior over losses (gains), respectively. Exhibit 6, Panel A graphs the value function in equation (10) for different curvature parameters. The lower the value of γ_1 and γ_2, the more pronounced the S-shape becomes over gains and losses. By itself, the value function is affected by extreme positive skewness, through the power utility specification over gains in equation (10)—which is the same mechanism we examined in the regular expected power utility function in Section 2. But, there is one more aspect of CPT that has a much bigger effect on CPT agents preferring large positive skewness lotteries, which we now discuss.

Probability Reweighting

An important part of the CPT framework is that individuals use transformed probabilities—which are decision weights because they may not sum to one. The probability reweighting in Tversky and Kahneman (1992) is taken separately over gains and over losses, but they both have the same shape:

$$w^+(p) = \frac{p^{\delta_1}}{(p^{\delta_1} + (1 - p^{\delta_1}))^{\frac{1}{\delta_1}}}, \tag{11}$$

where $w^+(p)$ is the weighting function over gains and

$$w^-(p) = \frac{p^{\delta_2}}{(p^{\delta_2} + (1 - p^{\delta_2}))^{\frac{1}{\delta_2}}}, \tag{12}$$

$w^-(p)$ is the weighting function over losses. Exhibit 6, Panel B graphs the weighting functions for the parameters taken by Tversky and Kahneman (1992), $\delta_1 = 0.61$ and $\delta_2 = 0.69$. These have inverted S-shaped functions and lie above the objective probability for small probabilities and below the objective probability for large probabilities. This has the effect that individuals overweight unlikely outcomes and underweight outcomes that occur more often.

[13] In addition, because the utility function is not globally concave, there is no guarantee that non-corner solutions can be found. (See the discussion by Ang, Bekaert, and Liu, 2005.) The Tversky and Kahneman (1992) values lead to finite solutions in the empirical calibration we adopt for BTC, equity, and bond returns as per Exhibit 3.

The decision weights π^+ and π^- for gains and losses, respectively, are given by:

$$\begin{aligned} \pi_1^- &= w^-(p_1), \pi_1^+ = w^+(p_n) \\ \pi_i^- &= w^-(p_1 + p_2 + \cdots + p_i) - w^-(p_1 + p_2 + \cdots + p_{i-1}) \text{ for } 1 < i \le k \\ \pi_j^+ &= w^+\left(p_j + p_{j+1} + \cdots + p_n\right) - w^+\left(p_{j+1} + p_{j+2} + \cdots + p_n\right) \text{ for } k < j < n \end{aligned} \tag{13}$$

where we have ordered the gains and losses $x_1 \le x_2 \le \cdots x_k \le 0 \le x_{k+1} \le x_{k+2} \le \cdots \le x_n$ over n states, with the zero reference point occurring between states k and $k+1$. The decision weight functions π^+ and π^- in equation (13) can be interpreted as the reweighted probabilities of outcomes *at least as good* as x_i for π^+ and *at least as bad* as x_i for π^-.[14]

Combining the value function in equation (10) with the decision weights in equation (13), we can define CPT utility, V, as:

$$V = \sum_{i=1}^{k} \pi_i^- \, v(x_i) + \sum_{j=k+1}^{n} \pi_j^+ \, v(x_j), \tag{14}$$

where the weighting functions (π^+ and π^- in equation (13)) and the value functions (v in equation (10)) take separate forms for negative losses and positive gains relative to the reference point.

The probability reweighting causes CPT to be very sensitive to skewness effects. CPT individuals overweight events with low objective probabilities, and the calibrated bliss regime in the mixture of Normals (see equation (1) and Exhibit 3) is a low probability outcome. Thus, the bliss states are assigned large decision weights. Even though agents are risk-averse over the large gain in the bliss regime, the payoff in the bliss regime is so large that an agent with CPT utility highly desires exposure to BTC right-tail skewness.

3.2 BTC Allocation under Cumulative Prospect Utility

We follow a similar exercise to what was done with power utility in Section 2. First, we compute the optimal equity weight in a two-asset equity-bond portfolio with the Tversky and Kahneman (1992) CPT parameters.

[14] Since we are using Gaussian-Hermite quadrature for the mixture of Normals distribution, the objective state space is naturally the Gaussian-Hermite weights, which are converted to decision weights following equation (13). A larger amount of quadrature points are required for accuracy for each asset than with power utility.

With the empirical fit of the mixture of Normals distribution for BTC (reported in Exhibit 3) and assuming the distributions of equity and bond returns are log normal, the optimal equity allocation in an equity-bond portfolio is 28%. We plot the CPT utility function in Exhibit 7, Panel A, which has a local maximum at this point.

Now, we hold fixed the equity-bond mix at 28-72 and solve for the BTC allocation. CPT is so sensitive to the large positive skew that the optimal allocation at the estimated parameter values for the empirical mixture of Normals distribution (equation (1) with parameters reported in Exhibit 3) is $+\infty$. This is extreme sensitivity to the small probability of a very large payoff, which under CPT receives a much higher weight than the real-world probability.

In order to obtain finite solutions, we change the mean of BTC in the normal regime to correspond to a loss of 90%. That is, we set $\exp(\mu_2) = 0.1$, or $\mu_2 = -2.303$. We also change the probability of the bliss regime to $p = 0.001$, with the same mean, μ_1, and standard deviation, σ_1, of the BTC bliss regime as the empirical estimate reported in Exhibit 3. This distribution has the same extreme right-hand tail payoff as the original process, but it occurs with a much smaller probability, $p = 0.001$, versus the empirical estimate of $p = 0.036$. Even with this tiny probability, the optimal BTC holding is 9.5% holding the 28-72 equity-bond portfolio fixed in pro-rata allocations. Exhibit 7, Panel B plots the CPT utility function for the three-asset BTC-equities-bonds portfolio. The maximum utlity corresponds to the optimal BTC holding of 9.5%, with the equity and bond holdings being held in the same 28-72 pro-rata allocation for the remaining 90.5% of the portfolio.

3.3 Varying the Probability of Bliss

In this section, we vary the probability of the bliss regime. We continue to hold the mean of the normal regime of BTC continuously compounded returns at $\mu_2 = -2.303$ with other parameters for the mixture of Normals processed unchanged from their empirical estimates (see Exhibit 3). We also hold fixed the log normal returns for equities and bonds at their empirical estimates.

Exhibit 8 reports the optimal BTC allocations under CPT. Panel A graphs the optimal BTC holding, h_{BTC}, as a function of the bliss state probability, p. The circles are the BTC holdings from the optimization and the dashed line is a fitted smoothed curve. There are discontinuities in the optimal BTC holdings due to the discrete sums in the definition of CPT utility (equations (10)-(14)) and our Gaussian-Hermite discretization, but the BTC holdings uniformly increase with p. The probabilities graphed are very small, from $p = 0.0001$ to $p = 0.0016$. To emphasize how small these probabilities are: we have used the annual horizon for our calibration so the bliss state only occurs once every 625 to 1,000 years! When the bliss probability is greater than 0.0004, the BTC allocation turns positive. At $p = 0.0005$, the optimal BTC holding is 1.3% and at $p = 0.0006$, the optimal BTC holding is 3.1%. The last point on the graph is $p = 0.0016$ which corresponds to a BTC holding of 17.8%. Only extremely tiny probabilities of BTC entering a state with large upside potential are required for behavioral investors with CPT utility to hold non-trivial amounts of BTC. It is worth noting that for probabilities of the bliss state greater than 0.0020, the optimal holding of BTC is $+\infty$.

This happens even when the unconditional mean of BTC is significantly negative. Panel B of Exhibit 8 graphs the implied mean of BTC (top graph) and the implied third central moment of BTC (bottom graph) as a function of p. We plot the unconditional continuously compounded means of BTC for the same values of the bliss probability in Panel A. These are less than -2.30, close to the value of μ_2 we have assumed corresponding to a loss of 90% (with $\exp(\mu_2) = 0.1$). The implied third central moments in Panel B are below 0.50, which are approximately a third of the empirical estimate of 1.44 (see Exhibit 1).

In summary, behavioral CPT investors are extremely sensitive to the right-hand skew of BTC returns. They allocate to meaningful amounts of BTC to portfolios with probabilities in the range of five or six in a thousand of a bliss regime occurring, even with unconditionally negative BTC returns.

4. Conclusion

Bitcoin (BTC) log returns exhibit an extremely large third central moment of 144% at the annual horizon, which is several orders of magnitude larger than the third central moments of stocks and bonds, which are -0.43% and 0.01%, respectively. We show that a mixture of Normals distribution to BTC returns can capture the large right-hand skew. There is one "normal" regime with relatively large volatility and lower conditional means. The other is a "bliss" regime that has an extremely high mean corresponding to price appreciation measured in hundreds of percent with a relatively smaller volatility. The bliss regime occurs with a small probability. The large positive skewness captured by the infrequently occurring bliss regime induces even investors with power utility, which is locally mean-variance, to hold significant allocations to BTC. Behavioral investors with Cumulative Prospect Theory (CPT) utility are even more sensitive to the large positive skew because they overweight the bliss regime compared to its objective probability. With CPT, an investor needs only to believe the bliss regime will occur with probabilities around 0.0005 to hold optimal BTC allocations of approximately 3%.

Our findings of the sensitivity to BTC allocations as a result of large positive skewness are valid for other asset classes that are positively skewed. Venture capital (VC) returns (see Cochrane, 2005) and individual stock returns (see Bessembinder, 2018) exhibit positive skewness and mixture of Normals distributions, together with preferences that capture higher moment effects, could also be useful for determining optimal holdings of those asset classes.

An important caveat is that we have taken only BTC returns and not examined the very large cross section of cryptos. This deserves separate attention because skewness is not a coherent measure of risk as defined by Artzner et al. (1999), so the third central moment properties of a diversified crypto portfolio could be *larger* than the individual third moments of each crypto. There are style factor strategies that can generate alpha in the cross-section of cryptos documented by Liu, Tsyvinski, and Wu (2019) and Liu, Liang, and Cui (2020) like a market crypto factor, size, and momentum, making an allocation to crypto more attractive. On the other hand, given the relatively short histories of crypto returns, survivorship bias may not be accurately measured (see Brown et al., 1992). It might be the case that the true probability of a bliss regime is lower than the empirical estimates we find, or lower than in the hypothetical comparative statics exercises we examine.

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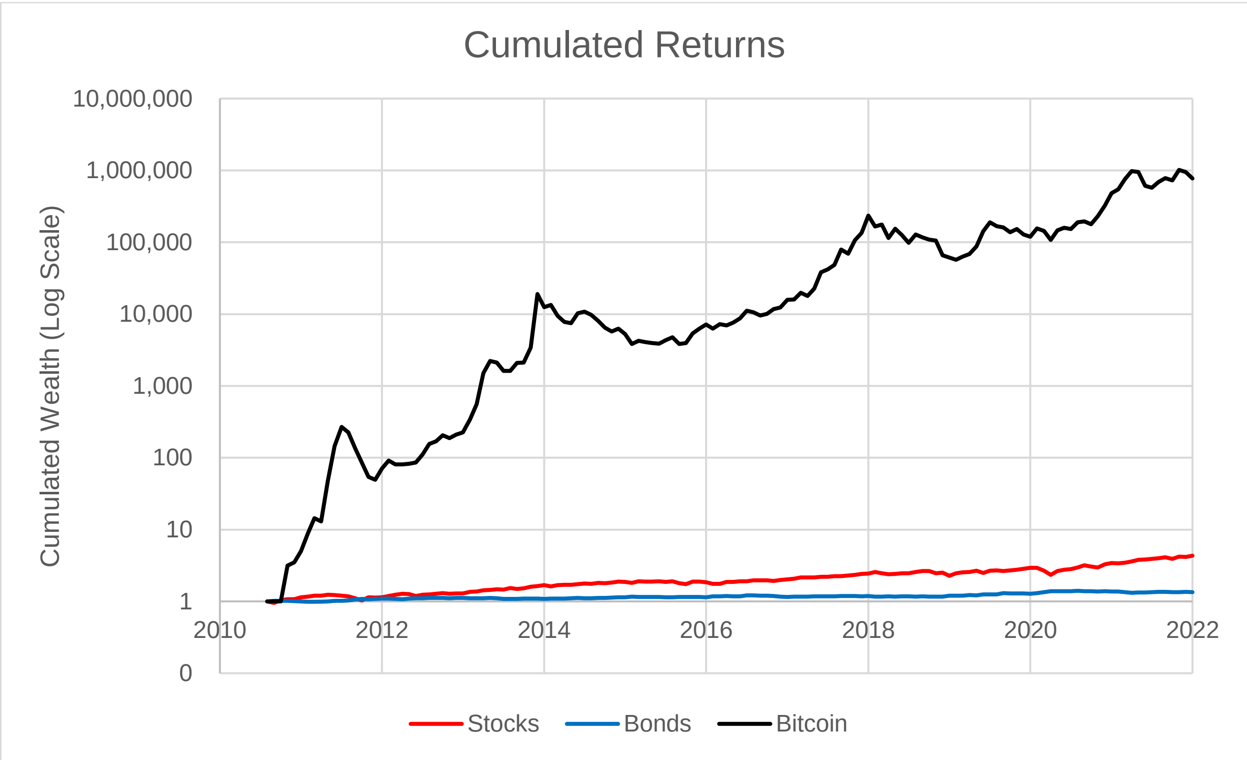
Exhibit 1: Summary Statistics

Panel A: Cumulated Returns



The figures shown relate to past performance. Past performance is not a reliable indicator of current or future results. Index performance returns do not reflect any management fees, transaction costs or expenses. Indices are unmanaged and one cannot invest directly in an index.
Sources: BlackRock with data from Refinitiv Datastream and Bloomberg, January 2022. Cumulated returns on a log scale are plotted for stocks (S&P 500), bonds (Bloomberg Barclays Treasury Index) and bitcoin from from July 2010 to December 2021. We use a $USD log scale on the y-axis.

Exhibit 1 Continued: Summary Statistics

Panel B: Estimates of Moments

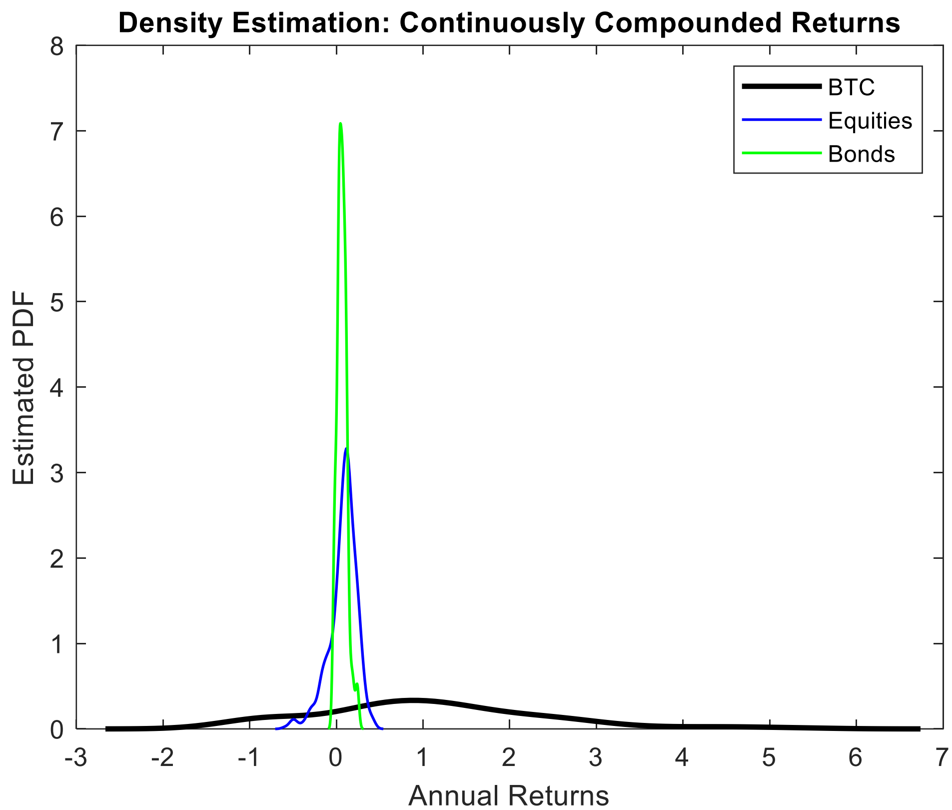
	Mean	Volatility	Sharpe Ratio	Third Central Moment	Skewness
BTC	1.0850	1.3224	0.8205	1.4389	0.6297
Equities	0.0769	0.1609	0.4779	-0.0043	-1.0417
Bonds	0.0669	0.0574	1.1668	0.0001	0.6929

Correlations

	BTC	Equities	Bonds
BTC	1.0000		
Equities	0.4112	1.0000	
Bonds	-0.2995	-0.0027	1.0000

Sources: BlackRock with data from Refinitiv Datastream and Bloomberg, January 2022. We report summary statistics of continuously compounded returns of bitcoin (BTC), stocks (S&P 500) and bond (Bloomberg Barclays Treasury Index) returns at the annual horizon. We use monthly frequency data at the annual horizon from July 2010 to December 2021 for BTC and from January 1973 to December 2021 for stocks and bonds. The univariate moments for each asset are computed using the longest available sample, and the correlation estimates are computed with the common sample across the assets.

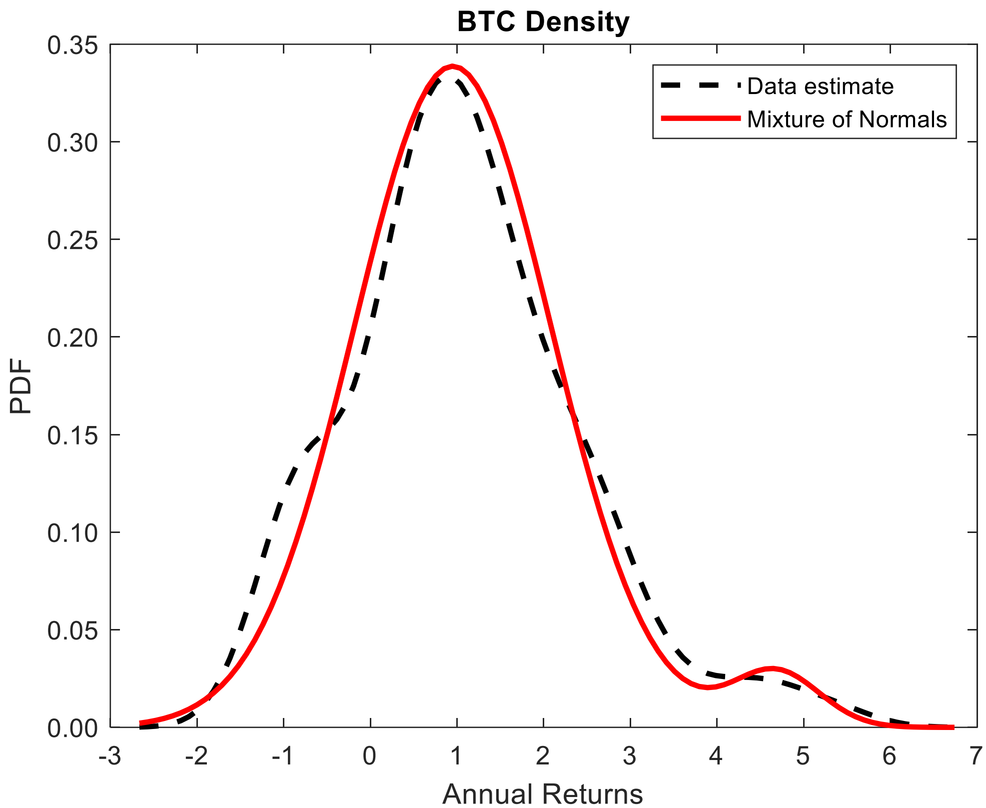
Exhibit 2: Probability Density Functions ("PDF")



Sources: BlackRock with data from Refinitiv Datastream and Bloomberg, January 2022. We plot probability density functions (PDFs) of continuously compounded returns of bitcoin (BTC), stocks (S&P 500) and bond (Bloomberg Barclays Treasury Index) returns at the annual horizon. We use monthly frequency data at the annual horizon from July 2010 to December 2021 for BTC and from January 1973 to December 2021 for stocks and bonds.

Exhibit 3: Mixture of Normals

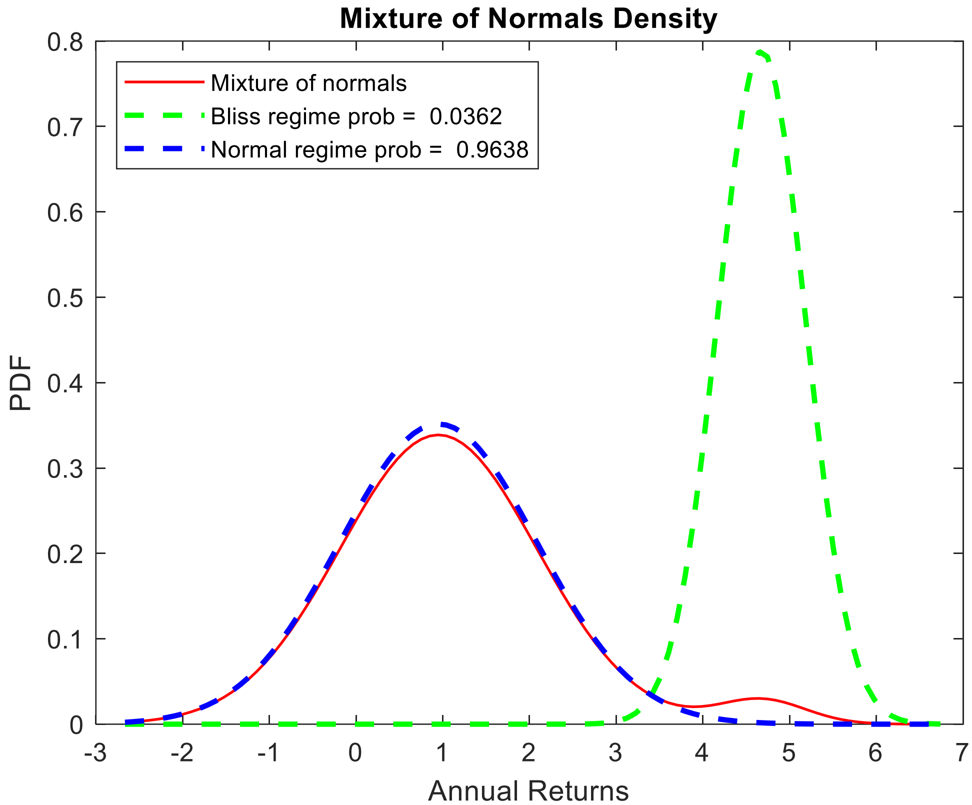
Panel A: Mixture of Normals Probability Density Function



Sources: BlackRock with data from Refinitiv Datastream and Bloomberg, January 2022. We plot the empirical probability density functions (PDF) of continuously compounded returns of bitcoin (BTC) returns at the annual horizon using monthly frequency data from July 2010 to December 2021. The empirical estimate is given by the dashed line. The red solid line represents the PDF of a fitted mixture of Normals distribution.

Exhibit 3 Continued: Mixture of Normals

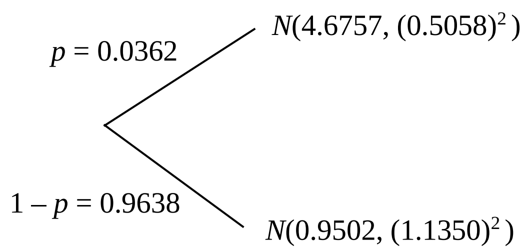
Panel B: Normal Distributions in Each Regime



Sources: BlackRock with data from Refinitiv Datastream and Bloomberg, January 2022. We plot the Normal distributions in each regime of the mixture of Normals distribution fitted to bitcoin (BTC) continuously compounded returns at the annual horizon using monthly frequency data from July 2010 to December 2021. The empirical estimates of the Normal distribution in each regime are given by the dashed lines. The red solid line represents the Normal distribution in the "normal" regime. The dashed green (blue) line represents the Normal distribution in the "bliss" ("normal") regime.

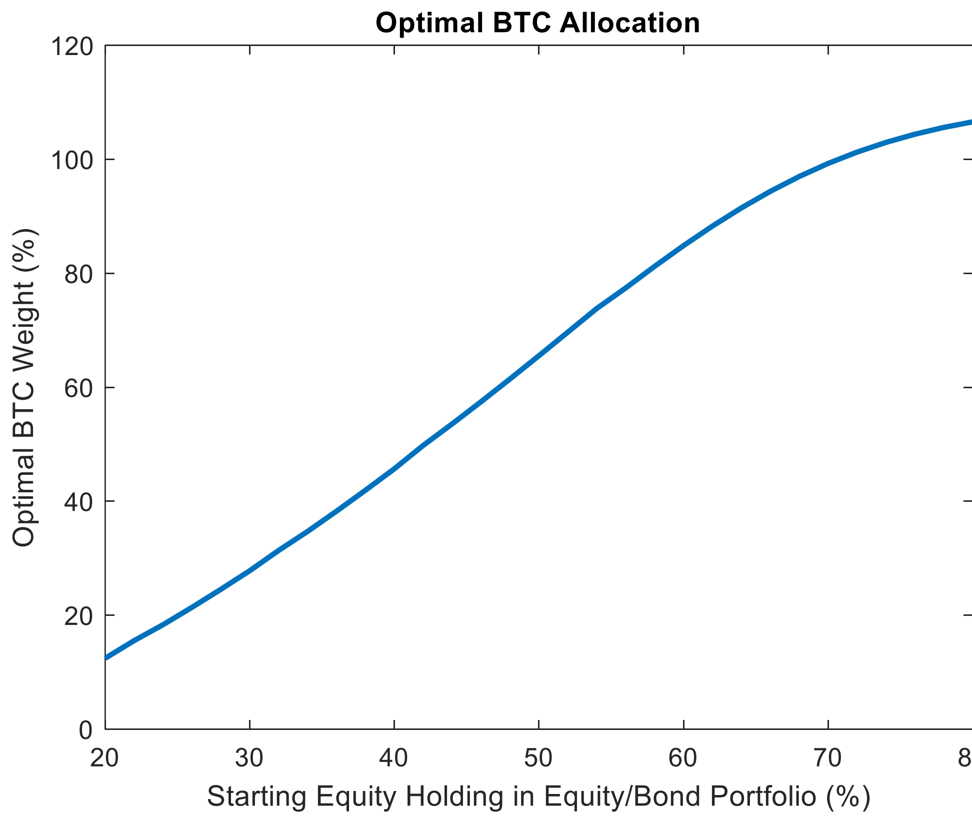
Exhibit 3 Continued: Mixture of Normals

Panel C: Mixture of Normal Distributions Parameters



Mixture of Normal distribution parameters in equation (1) for bitcoin (BTC) continuously compounded returns at the annual horizon using monthly frequency data from July 2010 to December 2021. The parameter p represents the probability of the "bliss" regime.

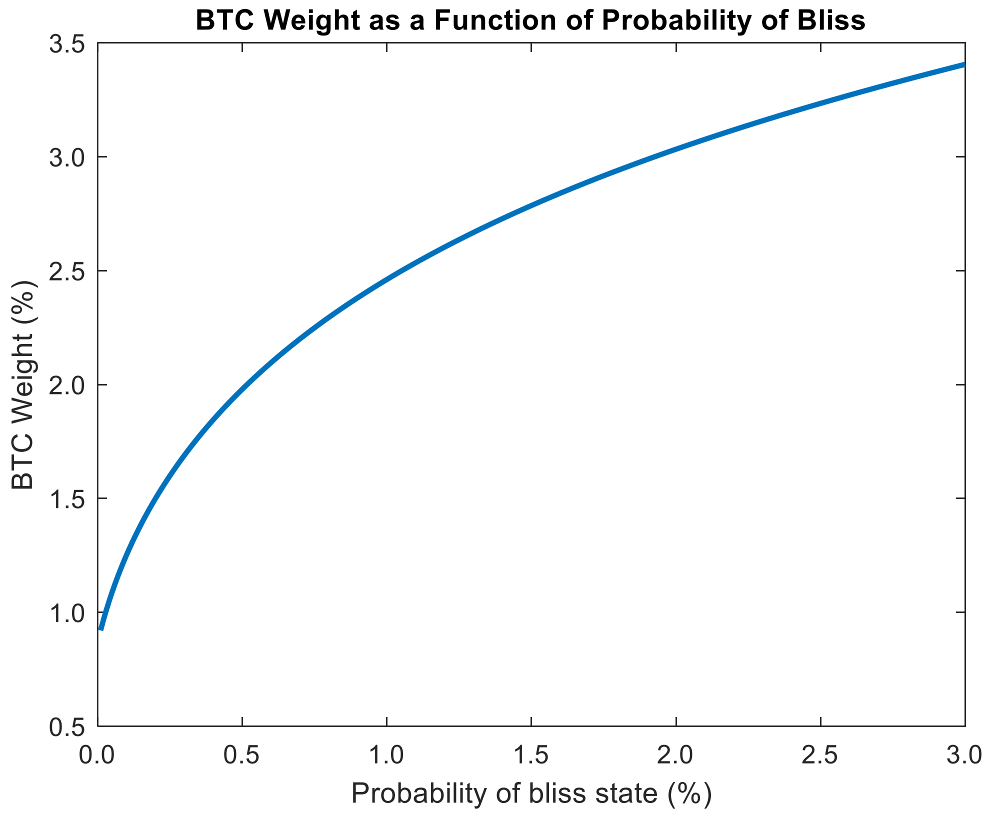
Exhibit 4: Allocations to BTC



Sources: BlackRock with data from Refinitiv Datastream and Bloomberg, January 2022. The graph presents the optimal BTC weight in a three asset portfolio of (BTC, equities, and bonds) with a starting equity position in a two asset portfolio (containing only equities and bonds) with power utility. For a given equity-bond mix corresponding to equity and bond weights of $h_{eq}$and h_{bd}, respectively, where $h_{eq} + h_{bd} = 1$, we compute the risk aversion parameter that yields the given equity-bond allocation (see equation (4)). The data generating process for equity and bonds is log normal. Then we add BTC to equities and bonds, where BTC follows the mixture of Normals distribution (as reported in Exhibit 3). The data sample used for the parameters is from January 1973 to December 2021. Holding fixed the risk aversion parameter from the two-asset problem, we compute the optimal BTC holding, h_{BTC}, assuming that the equity and bond weights are held in the proportions $(1 - h_{BTC})h_{eq}$ and $(1 - h_{BTC})h_{bd}$, respectively.

Exhibit 5: Sensitivity to Probability of Bliss

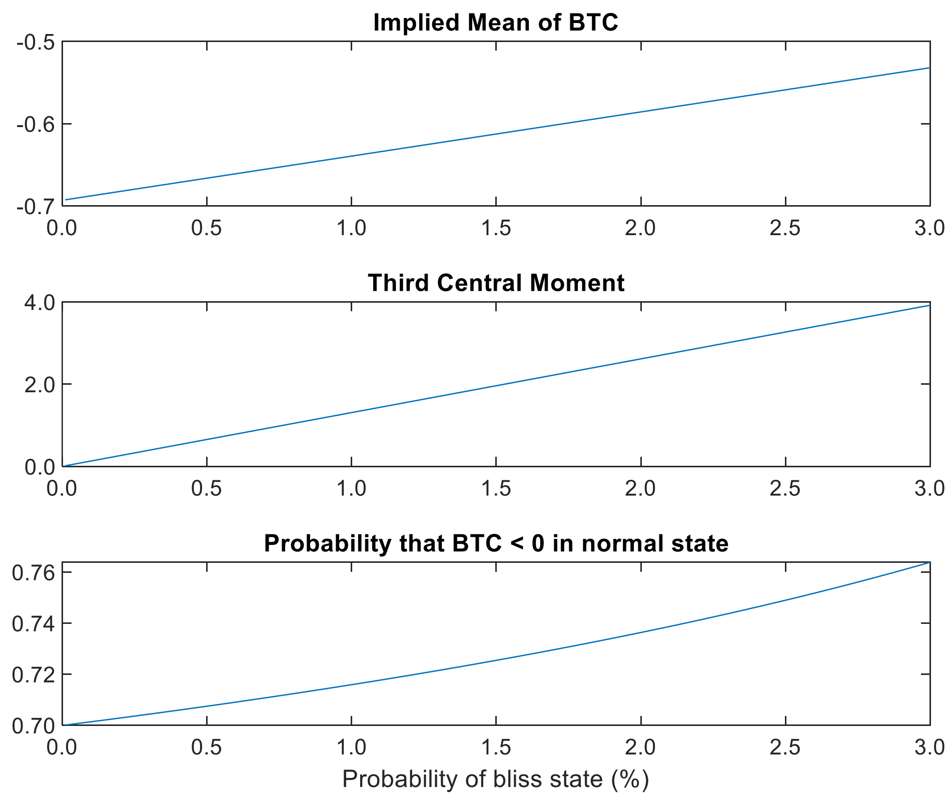
Panel A: Varying the Probability of Bliss



Sources: BlackRock with data from Refinitiv Datastream and Bloomberg, January 2022. For a given probability of the bliss state in the mixture of Normals distribution for BTC, we graph the resulting optimal BTC holding, h_{BTC}. We assume that the equity and bond weights are $(1 - h_{BTC})h_{eq}$ and $(1 - h_{BTC})h_{bd}$, respectively, with $h_{eq} = 0.6$ and $h_{bd} = 0.4$. We assume equity and bond returns are log normal and BTC follows the mixture of Normals distribution as reported in Exhibit 3, except that we set the mean of BTC in the normal regime to be $\mu_2 = -0.693$. The data sample used for the parameters is from January 1973 to December 2021.

Exhibit 5 Continued: Sensitivity to Probability of Bliss

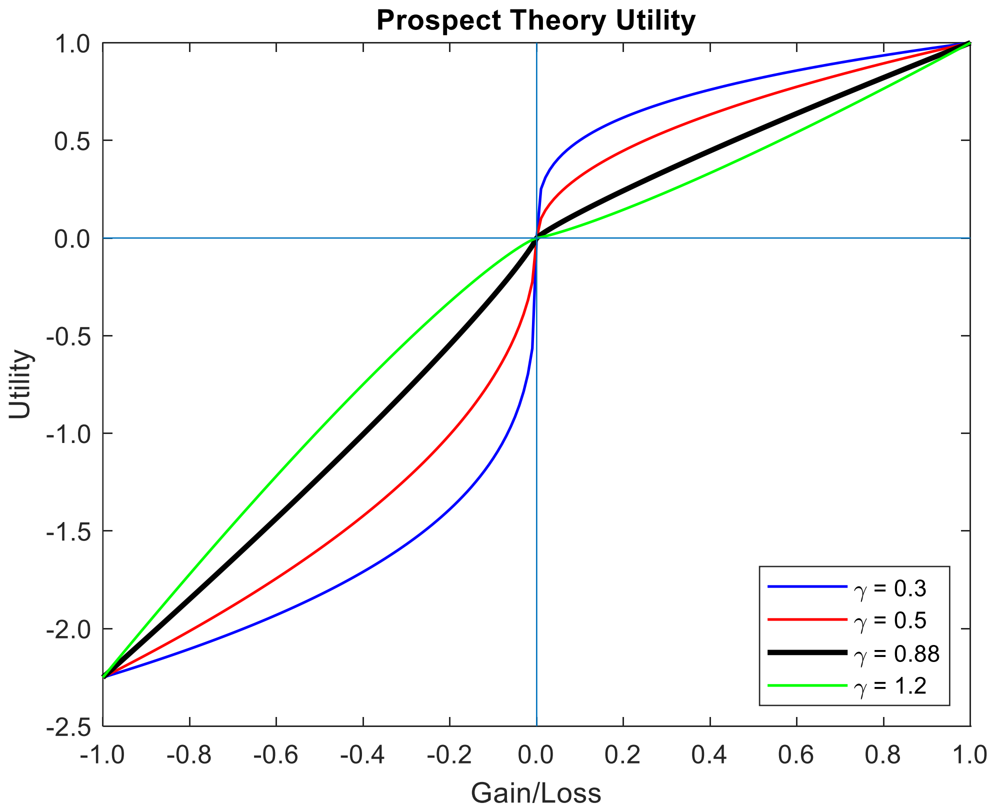
Panel B: Implied Statistics with the Probability of Bliss



Sources: BlackRock with data from Refinitiv Datastream and Bloomberg, January 2022. For the same probability of the bliss state in Panel A with the same parameters, we plot the implied unconditional mean of BTC in the top panel, the implied third central moment in the middle panel, and the probability that $\exp(r_{BTC}) < 1$ in the normal regime. The data sample used for the parameters is from January 1973 to December 2021.

Exhibit 6: Cumulative Prospect Theory

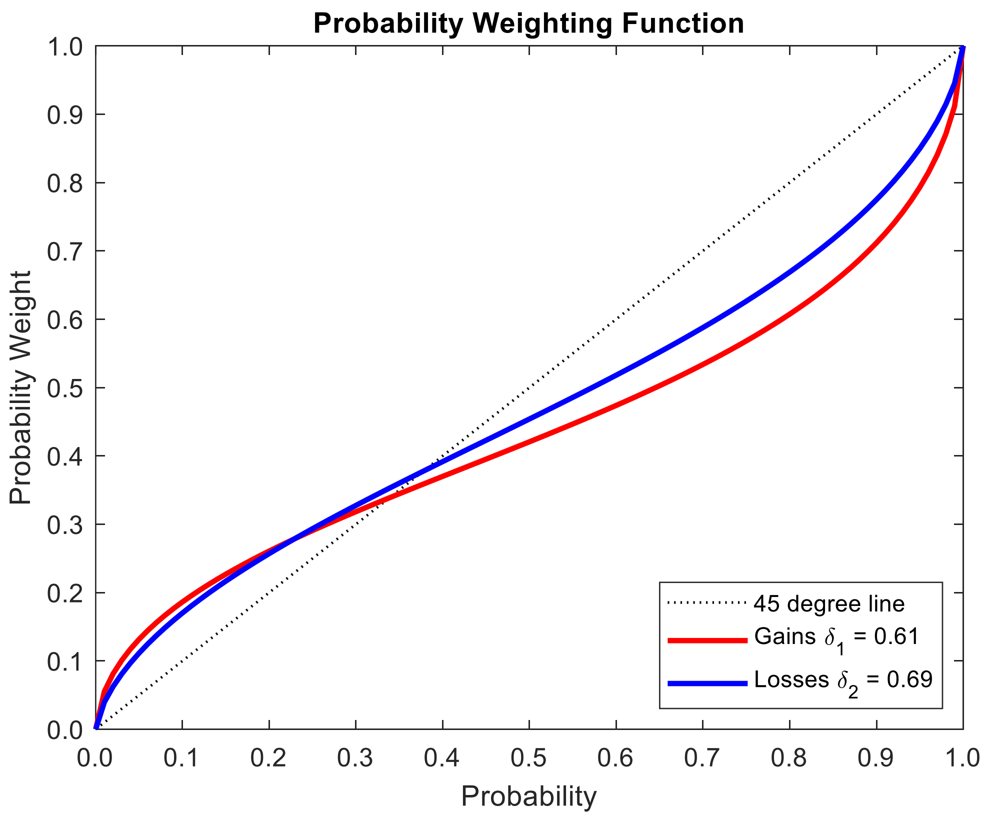
Panel A: Risk Seeking Over Losses, Risk Averse Over Gains



Sources: BlackRock. We plot the Cumulative Prospect Theory value function in equation (10) with the Tversky and Kahneman (1992) parameters of $\gamma = \gamma_1 = \gamma_2 = 0.88$ over gains and losses, x. The reference point corresponds to $x = 0$.

Exhibit 6 Continued: Cumuative Prospect Theory

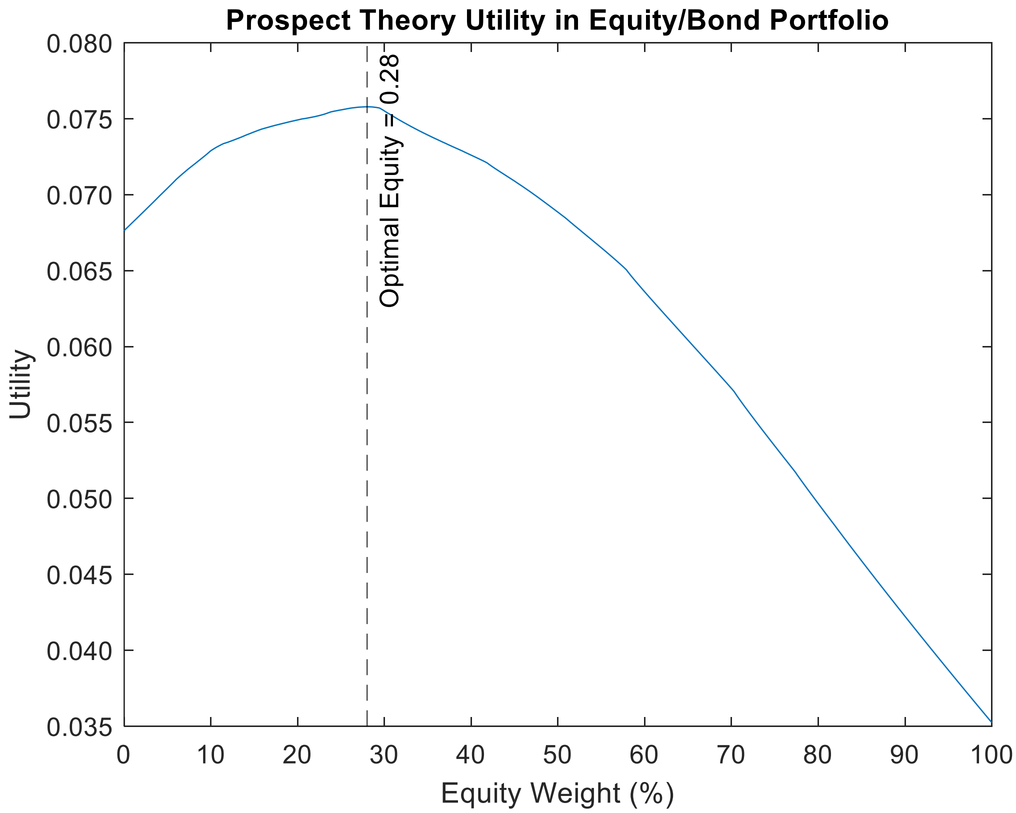
Panel B: Probability Weighting Function



Sources: BlackRock. We plot the Cumulative Prospect Theory probability reweighting functions $w^+(p)$ and $w^-(p)$ in equations (11) and (12) for gains and losses, respectively. We use the Tversky and Kahneman (1992) parameters of $\delta_1 = 0.61$ and $\delta_2 = 0.69$.

Exhibit 7: Cumulative Prospect Utility in Portfolios

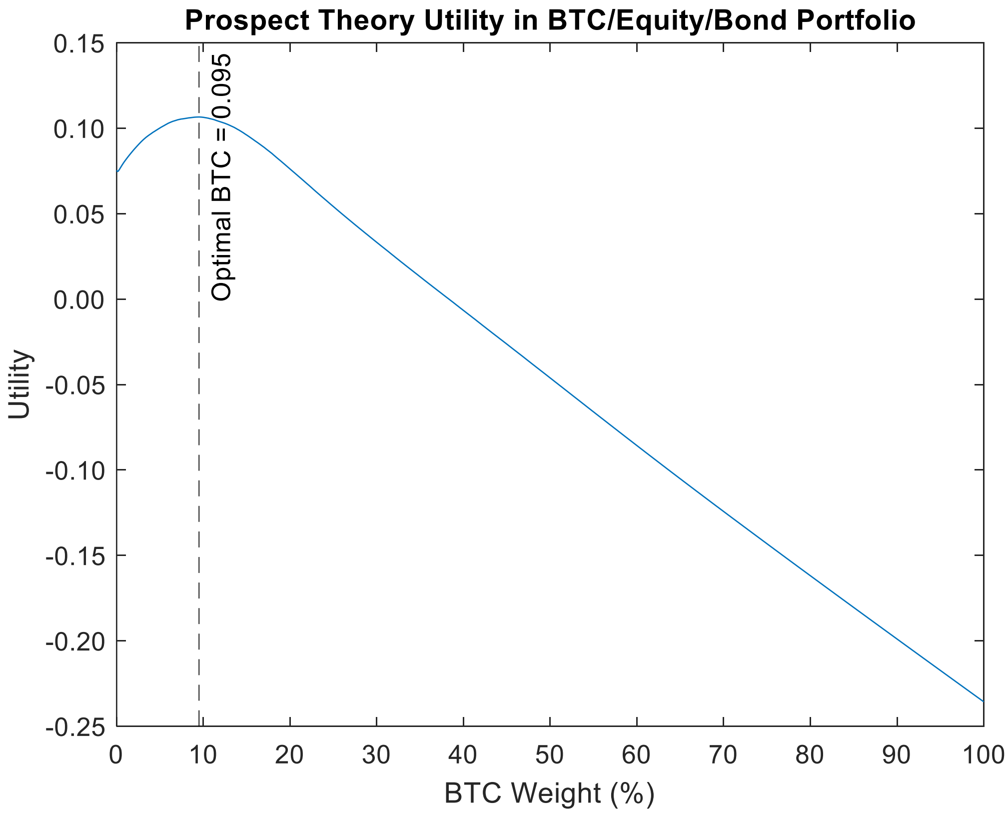
Panel A: Equity-Bond Portfolio



Sources: BlackRock with data from Refinitiv Datastream and Bloomberg, January 2022. The figure plots the CPT utility function given in equations (10)-(14) for an equity-bond portfolio assuming the equity and bond returns are jointly log normally distributed calibrated to the moments listed in Exhibit 1, Panel B. The data sample used for the parameters is from January 1973 to December 2021.We use the Tversky and Kahneman (1992) parameters of $\gamma = \gamma_1 = \gamma_2 = 0.88$, $\delta_1 = 0.61$ and $\delta_2 = 0.69$, $\lambda = 2.25$, and a reference point of zero.

Exhibit 7 Continued: Cumulative Prospect Utility in Portfolios

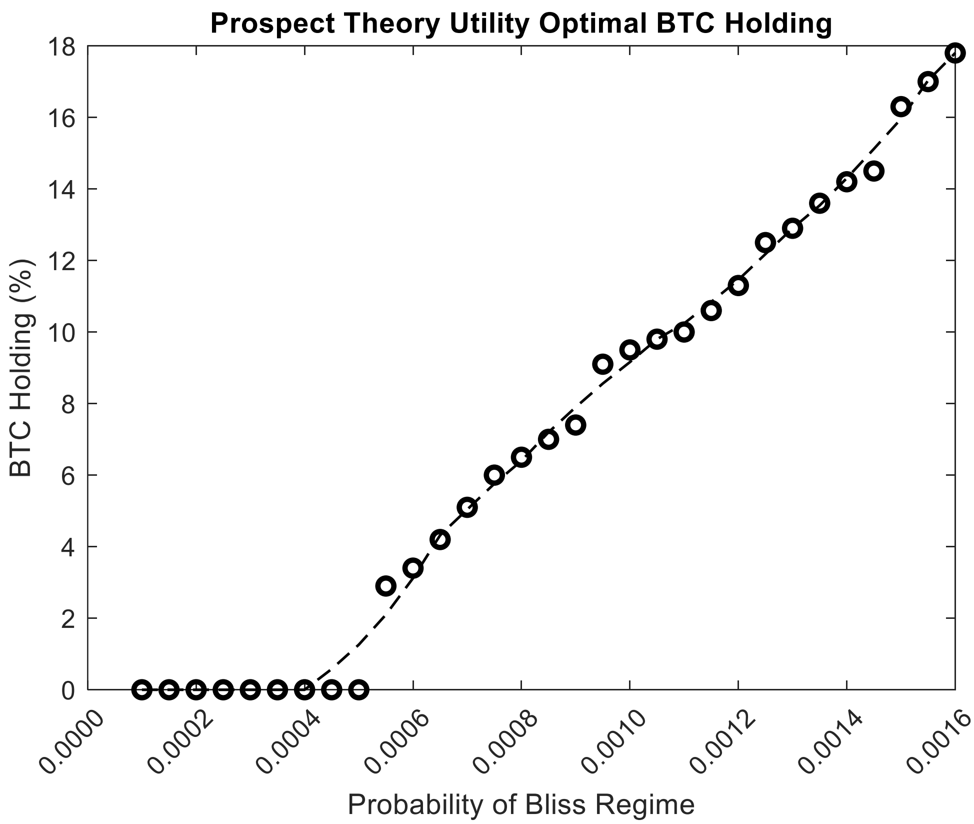
Panel B: BTC-Equity-Bond Portfolio



Sources: BlackRock with data from Refinitiv Datastream and Bloomberg, January 2022. The figure plots the CPT utility function given in equations (10)-(14) for a BTC-equity-bond portfolio assuming that BTC follows a mixture of Normals distribution (with parameters reported in Exhibit 3) and equity and bond returns are jointly log normally distributed calibrated to the moments listed in Exhibit 1, Panel B. The data sample used for the parameters is from January 1973 to December 2021. We use the Tversky and Kahneman (1992) parameters of $\gamma = \gamma_1 = \gamma_2 = 0.88$, $\delta_1 = 0.61$ and $\delta_2 = 0.69$, $\lambda = 2.25$, and a reference point of zero.

Exhibit 8: BTC Allocation under Cumulative Prospect Theory and the Probability of Bliss

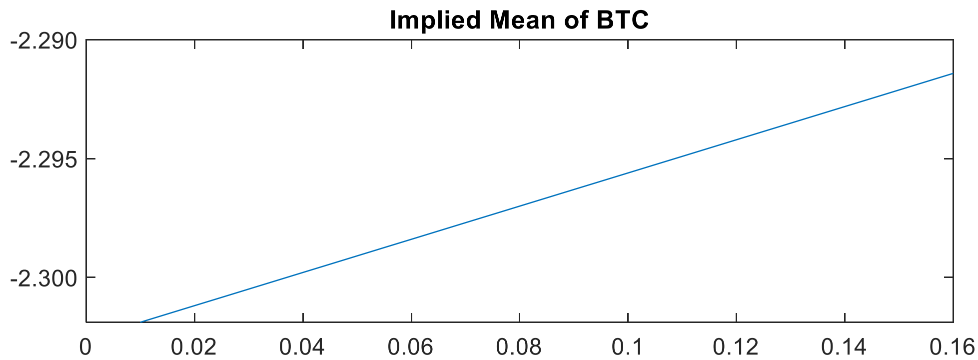
Panel A: Varying the Probability of the Bliss Regime

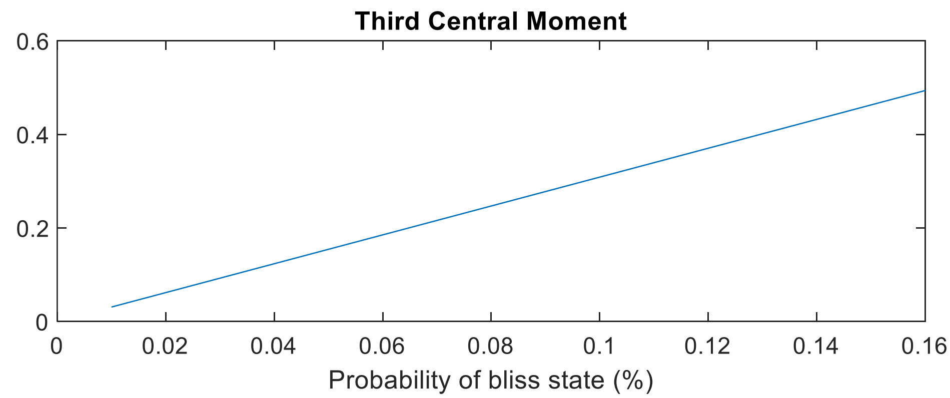


Sources: BlackRock with data from Refinitiv Datastream and Bloomberg, January 2022. We plot optimal BTC holdings for a CPT investor given in equations (10)-(14) for a BTC-equity-bond portfolio as we vary the probability of the bliss regime, p. The circles are the BTC holdings from the optimization and the dashed line is a fitted smoothed curve. We assume that the mean of the continuously compounded BTC normal regime, $\mu_2 = -2.303$. Other parameters for the BTC process are held fixed at their values reported in Exhibit 3. The data sample used for the parameters is from January 1973 to December 2021. Equity and bond returns are jointly log normally distributed with parameters calibrated to the moments listed in Exhibit 1, Panel B. We use the Tversky and Kahneman (1992) parameters of $\gamma = \gamma_1 = \gamma_2 = 0.88$, $\delta_1 = 0.61$ and $\delta_2 = 0.69$, $\lambda = 2.25$, and a reference point of zero.

Exhibit 8 Continued: BTC Allocation under Cumulative Prospect Theory and the Probability of Bliss

Panel B: Implied Statistics with the Probability of Bliss





Sources: BlackRock with data from Refinitiv Datastream and Bloomberg, January 2022. For the same probability of the bliss state in Panel A with the same parameters, we plot the implied unconditional mean of BTC in the top panel and the implied third central moment in the bottom panel. The data sample used for the parameters is from January 1973 to December 2021.

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